# The Science and Practice of Trend-following Systems


**Artur Sepp**

Global Head of Investment Services Quant Group, LGT Bank



CQF Volatility and Risk in Quant Finance Conference

4 June 2025

**Content**

1. Design of Trend-Following (TF) Systems

2. Expected returns of European TF system under white noise, auto-regressive and fractional processes with long memory

3. Empirical analysis

4. TF for diversification of long-only portfolios

- Sepp A., Lucic V. (2025) The Science and Practice of Trend-following Systems, SSRN, https://ssrn.com/abstract=3167787

**Overview**

• TF Systems (CTAs, Managed Futures) are systematic strategies trading in futures markets

1) **European TF** systems from European CTAs: CFM *[Bouchaud2017]*, AHL *[Baz2015]*

2) **American TF** systems originate from "Turtle Traders" in 1983 (*[Faith2007]*)

3) **Time Series Momentum (TSMOM)** systems originate from academic studies (*[Moskowitz2012]*)

**Quantitative analysis of profitability of European TF system**

i) Guassian framework: *[Grebenkov2014]*, *[Zakamulin2020]*, *[Martin2023]*, *[Valeyre2025]*

ii) Spread between long- & short-term realised variances: *[Bouchaud2017]*

- **Our contribution**:

1) Formulas for expected returns of European TF system under generic process for futures returns

2) Formulation & empirical consistency of three types of TF systems

**European Trend-Following System**

- $\{s_t\}$ is continuous prices of a futures contract with lag-1 difference $d_t$ and relative return $r_t$:

$$d_t = s_t - s_{t-1},\ r_t = s_t/s_{t-1} - 1 \tag{1}$$

- *[Bouchaud2017]*, *[Baz2015]* specify weight:

$$w_t = \frac{1}{\sigma_t}\mathcal{L}^{(\nu)}\left(\frac{d_t}{\sigma_{t-1}}\right) \tag{2}$$

- $\mathcal{L}^{(\nu)}$ is EWMA filter with smoother $\nu$, $0 < \nu < 1$:

$$\mathcal{L}^{(\nu)}(z_t) = (1-\nu)\, z_t + \nu\mathcal{L}^{(\nu)}(z_{t-1}) \tag{3}$$

$\sigma_t$ is EWMA volatility: $\sigma_t = \sqrt{\mathcal{L}^{(\nu_\sigma)}\left(d_t^2\right)}$

**American Trend-Following Systems**

- American TF systems not studied in literature extensively (sketches in *[Curtis2007]*, *[Covel2009]*)

- Average true range (ATR) as volatility measure

- A long position is initiated if the fast filter exceeds the slow filter plus a buffer ($\nu_1 > \nu_2$):

$$\mathcal{L}^{(\nu_2)}(s_t) > \mathcal{L}^{(\nu_1)}(s_t) + q \times ATR_t \qquad (4)$$

where $q$ is the width of entry-point buffer

**Position Sizing in American TF System**

- The position size $p^{long}$ and the stop-loss:

$$p^{long} = r\frac{s_t}{ATR_t}, \;\; stop\text{-}loss_t^{long} = s_t - p \times ATR_t \quad (5)$$

where $r$ is risk multiple

$p$ is scale of stop-loss buffer

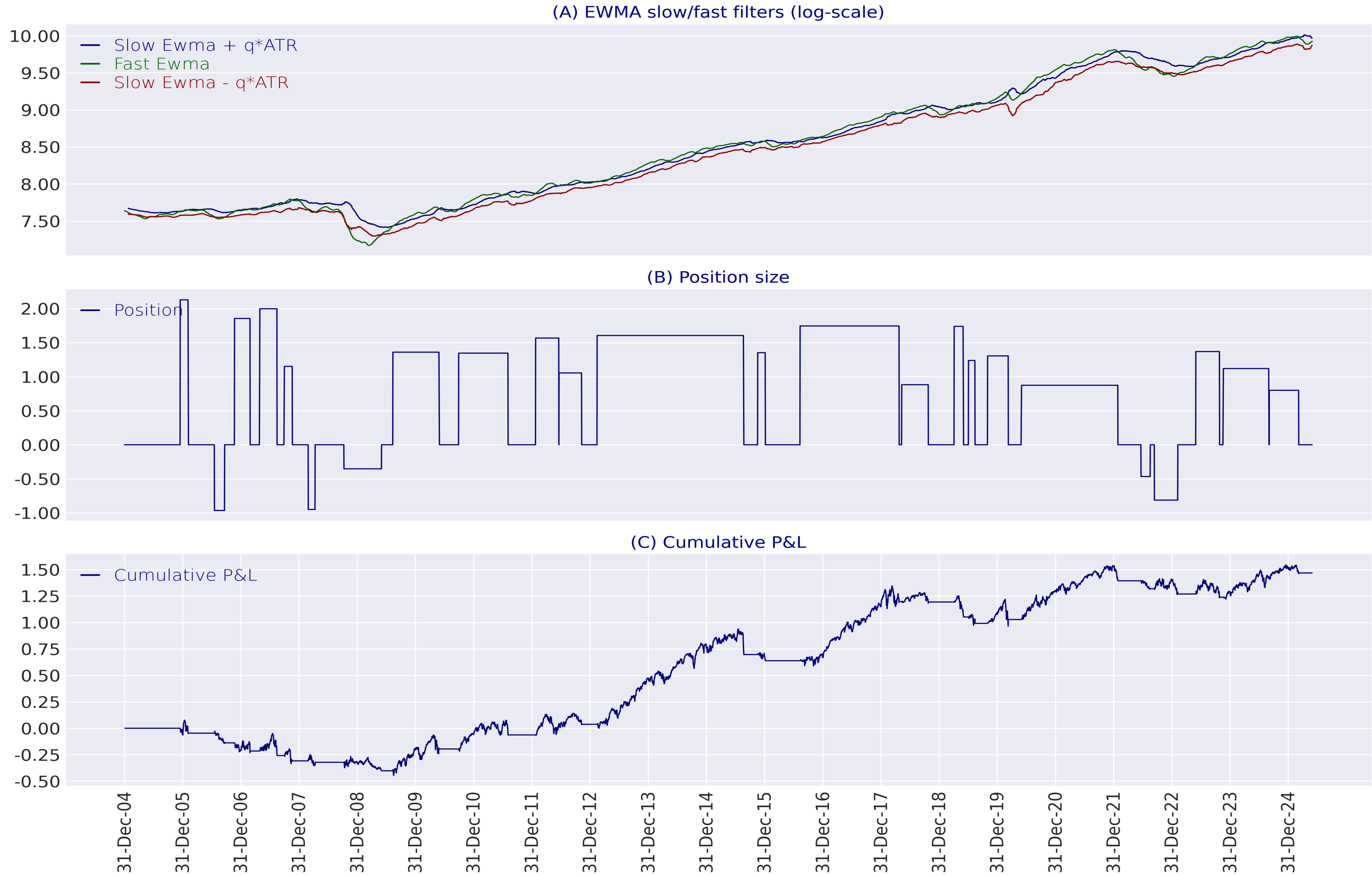
- The long position is closed if stop-loss is breached:

$$s_t < stopp\text{-}loss_t^{long} \quad (6)$$

otherwise, the stop-loss is updated:

$$stop\text{-}loss_t^{long} = \max\left\{stop\text{-}loss_t^{long}, s_t - pATR_t\right\}$$

# American TF system for Nasdaq index future



**Time Series Momentum (TSMOM) System**

• TSMOM (*[Moskowitz2012]*, *[Hurst2013]*, *[Baltas2015]*) and risk-adjusted TSMOM (*[Dudler2015]*) use 12 rolling returns:

$$w_t = \frac{1}{M}\sum_{t=1}^{M} sign\left(\frac{s_t}{s_{t-12}} - 1\right)\frac{\sigma_{target}}{\sigma_t^{an}} \qquad (7)$$

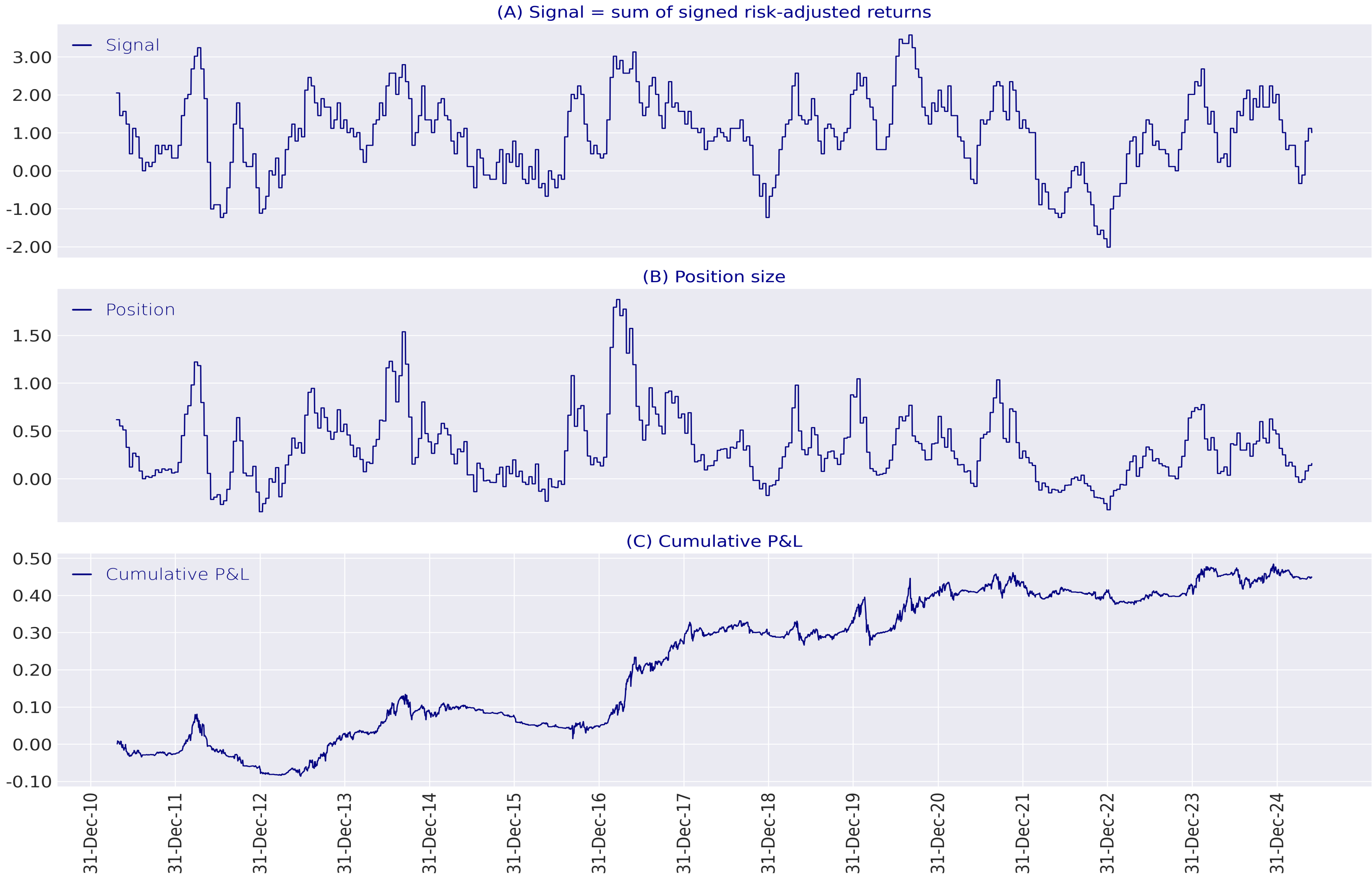
where: $sign(x)$ is sign function

$\{s_t\}$ is set of month end prices

$\sigma_{target}$ is annualised volatility target

$\sigma_t^{an}$ is annualised volatility estimate

# TSMOM system for Nasdaq index future



**European TF System using EWMA Filter**

- EWMA filter (*[Holt1957]*):

$$\begin{aligned}\mathcal{L}^{(\nu)}(z_t) &\equiv (1-\nu)\left(z_t + \nu^1 z_{t-1} + \nu^2 z_{t-2} + ...\right) \\ &= (1-\nu)\, z_t + \nu \mathcal{L}^{(\nu)}(z_{t-1})\end{aligned} \tag{8}$$

- Smoothing parameter $\nu$, $0 < \nu < 1$:

$$\nu = 1 - \frac{2}{span + 1} \tag{9}$$

where $span$ is the lookback period

- The variance of EWMA filter for serially independent variables $\{z_t\}$ with variance $\vartheta_0$ is given by:

$$\mathbb{V}ar\left[\mathcal{L}^{(\nu)}(z_n)\right] = \frac{1}{span}\vartheta_0 \tag{10}$$

**Variance-Preserving & Long-Short EWMA Filters**

- Variance-preserving EWMA filter:

$$\tilde{\mathcal{L}}^{(\nu)}(z_t) = \sqrt{\frac{1+\nu}{1-\nu}}\mathcal{L}^{(\nu)}(z_t) \tag{11}$$

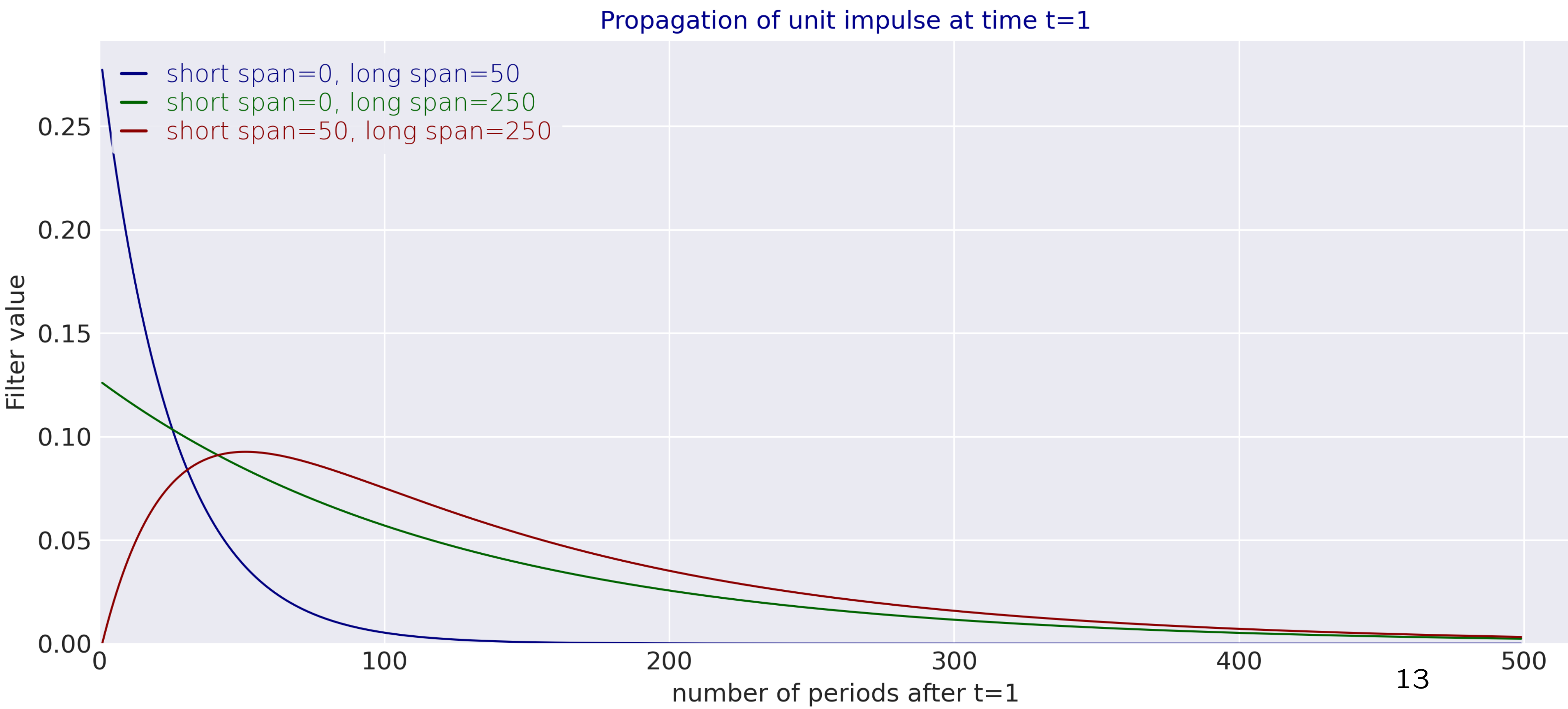
- Variance-preserving Long-Short EWMA filter:

$$\begin{aligned}\widetilde{\mathcal{LS}}^{(\nu_1,\nu_2)}(z_n) &\equiv \tilde{l}_1\widetilde{\mathcal{L}}^{(\nu_1)}(z_n) - \tilde{l}_2\widetilde{\mathcal{L}}^{(\nu_2)}(z_n) \\ &\equiv l_1\mathcal{L}^{(\nu_1)}(z_n) - l_2\mathcal{L}^{(\nu_2)}(z_n)\end{aligned} \tag{12}$$

with loadings $l_1$, $l_2$ and $\tilde{l}_1$, $\tilde{l}_2$

## Illustration of Long-Short EWMA Filter

- Example: propagation of unit impulse using $\widetilde{\mathcal{LS}}^{(\nu_1,\nu_2)}(z_n)$ with $\{z_t\} = (1, 0, 0, ...)$
$\nu_1 = 1 - 2/(long\ span + 1), \nu_2 = 1 - 2/(short\ span + 1)$



## European Trend-Following System

1. Daily returns $\{r_t\}$ with volatilities $\{\sigma_t\}$
2. Volatility-normalized daily returns $z_t$:

$$z_t = \frac{r_t}{\sigma_{t-1}} \tag{13}$$

3. Signal $s_t$ is computed with either var-preserving EWMA in Eq (11) or long-short filter in Eq (12):

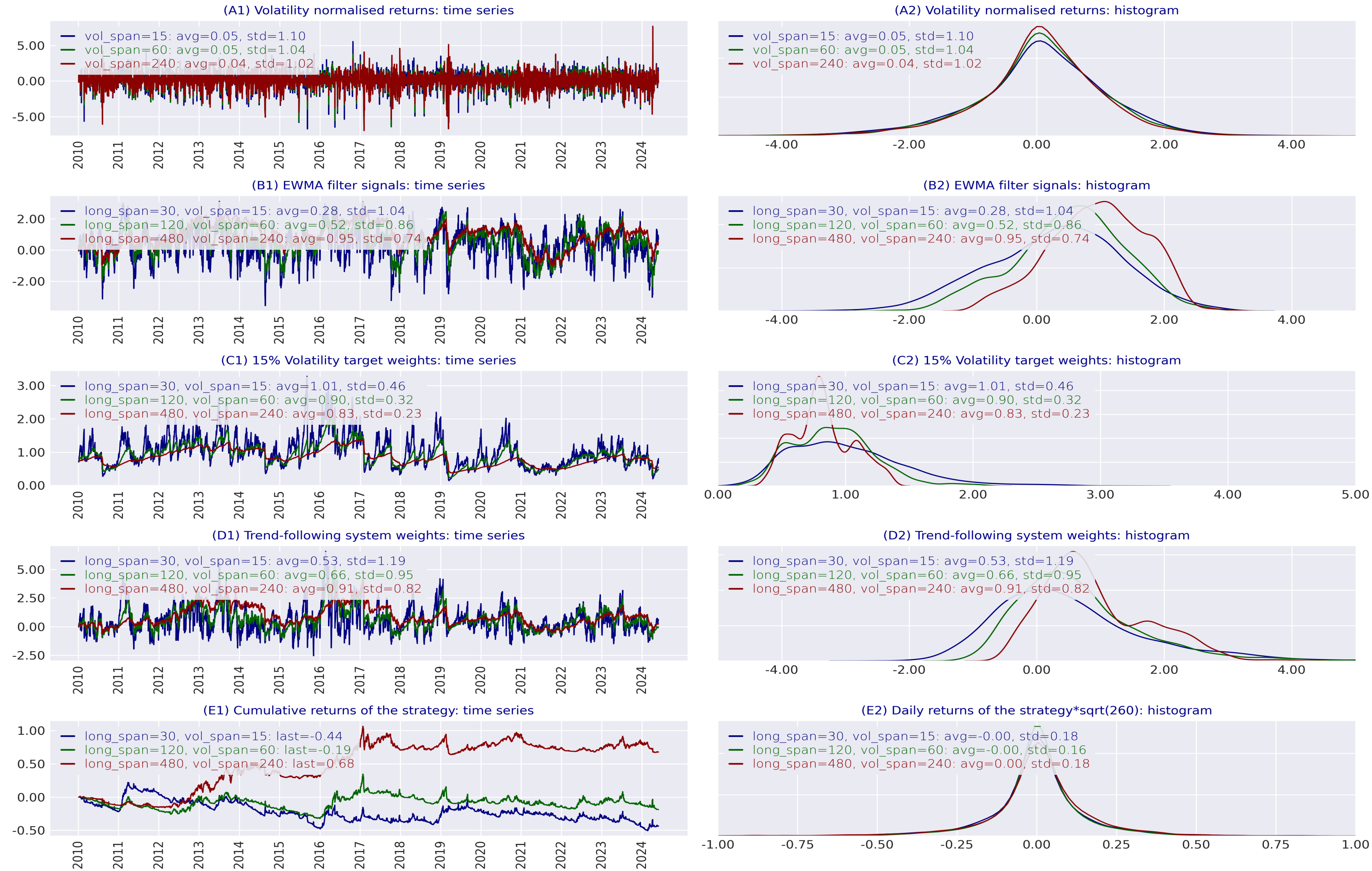
$$s_t = \widetilde{\mathcal{L}}^{(\nu)}(z_t),\ s_t = \widetilde{\mathcal{LS}}^{(\nu_1,\nu_2)}(z_t) \tag{14}$$

4. Position size:

$$w_t = s_t v_t,\ v_t = \frac{\sigma_{target}}{\sqrt{a}\sigma_t} \tag{15}$$

where $\sigma_{target}$ is vol-target & $a = 260$

# European TF for Nasdaq index future with vol target 15%



**Realised Cumulative Return of European TF System**

- Cumulative return $F_T$ over period $T$:

$$F_T = \sum_{t=1}^{T} f_t \tag{16}$$

- The cumulative return of European TF system:

$$F_T = \frac{l\sigma_{target}}{\nu\sqrt{a}} E(T)$$

$$E(T) = (1-\nu) \sum_{m=1}^{\infty} \nu^m \sum_{t=1}^{T} (z_t - \overline{z}_T)(z_{t-m} - \overline{z}_T) + \nu T(\overline{z}_T)^2$$

where $\overline{z}_T$ is the sample mean $\overline{z}_T = \frac{1}{T}\sum_{t=1}^{T} z_t$

**Expected Cumulative Return of European TF**

- Consider a generic process for futures returns $r_t$ under statistical measure $\mathbb{P}$:

$$\begin{aligned} &\mathbb{E}^{\mathbb{P}}\left[r_t\right] = \mu, \ \mathbb{V}ar^{\mathbb{P}}\left[r_t\right] = \vartheta \\ &\mathbb{E}^{\mathbb{P}}\left[\frac{1}{\vartheta}(r_t - \mu)(r_{t-m} - \mu)\right] = \rho(m), \ m = 0, 1, ... \end{aligned} \tag{17}$$

- Expected annual return of European TF system in Eq (16) under $\mathbb{P}$ with $T = 1y$, $\vartheta = 1$, $\mu_{an} = a\mu$:

$$\begin{aligned} \bar{F}_{1y} = &\left(l\sigma_{target}\sqrt{a}\frac{1-\nu}{\nu}\right)\left[\sum_{m=0}^{\infty}\nu^m\rho(m) - 1\right] \\ &+ \left(\frac{l\sigma_{target}}{\sqrt{a}}\right)\mu_{an}^2 \end{aligned} \tag{18}$$

**Turnover Analysis**

- Futures vols range from 2% for STIR to 30%+ for commodities

- Volatility-adjusted annualised turnover $U_t$ is stable measure:

$$U_t \equiv \sqrt{a}\sigma_t \left|w_t - w_{t-1}\right| \tag{19}$$

where $\sigma_t$ is vol of daily returns and $a = 260$

- Assuming serially independent $z_t$, the expected turnover $U_t$ is given in the leading order by:

$$\mathbb{E}\left[aU_t\right] = \frac{2a}{\sqrt{\pi}}\sigma_{target}\sqrt{1-\nu} \tag{20}$$

**Analysis of Expected Return under Specific Returns Dynamics**

**1)** Autocorrelation profile

**2)** Mean/drift of the process for generating returns

• For verification of analytical results, generate $10K$ of Monte Carlo paths with 100 years of daily returns

• Compute MC estimates of annual return of European TF $\{\widehat{F}_{1y}\}$ and confidence $1.96 stdev(\widehat{F}_{1y})/\sqrt{10000}$

• Volatility target of instruments is 15%

• Span of EWMA filter in days:
$\{1w : 5,\ 2w : 10,\ 1m : 22,\ 3m : 65,\ 6m : 130,\ 1y : 260,\ 2y : 520\}$

**Expected Annual Return for White Noise Process**

$$r_t = \mu + \epsilon_t \tag{21}$$

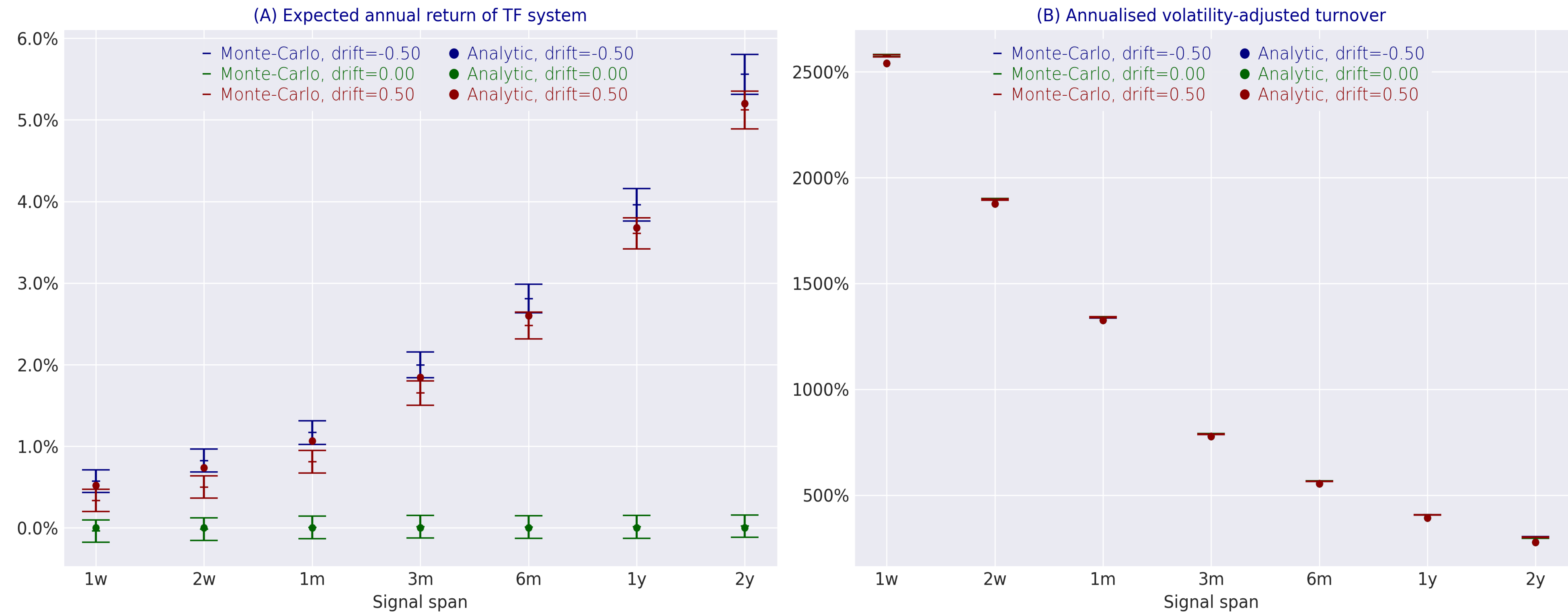
where $\epsilon_t$ is normal with var $1/a$ & mean $\mu_{an} = a\mu$

- (A) Auto-correlation is zero, $\rho_T(m) = 0$, $m > 1$
- The expected annual return $\overline{F}_{1y}$ in Eq (18):

$$\bar{F}_{1y} = \left(\frac{l\sigma_{target}}{\sqrt{a}}\right)\mu_{an}^2 \tag{22}$$

## Expected Annual Return for White Noise Process

- Panels (B) & (C): analytical values (dots) and 95% Confidence intervals of Monte Carlo estimates



**Expected return for AR-1 process with zero drift**

$$r_t = \mu + \phi r_{t-1} + \epsilon_t \tag{23}$$

where $|\phi| < 1$, $\epsilon_t$ is normal with var $1/a$ & mean $\mu_{an} = a\mu$

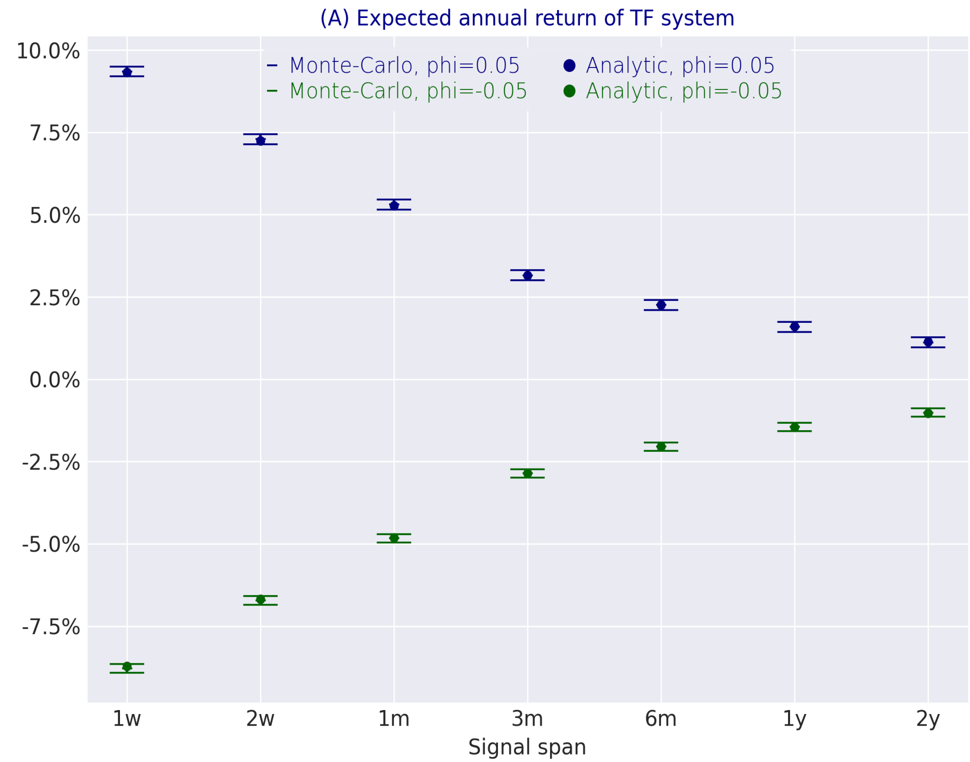
- The auto-correlation of AR-1: $\rho_T(m) = \phi^m$
- Expected 1y of European TF in Eq (18):

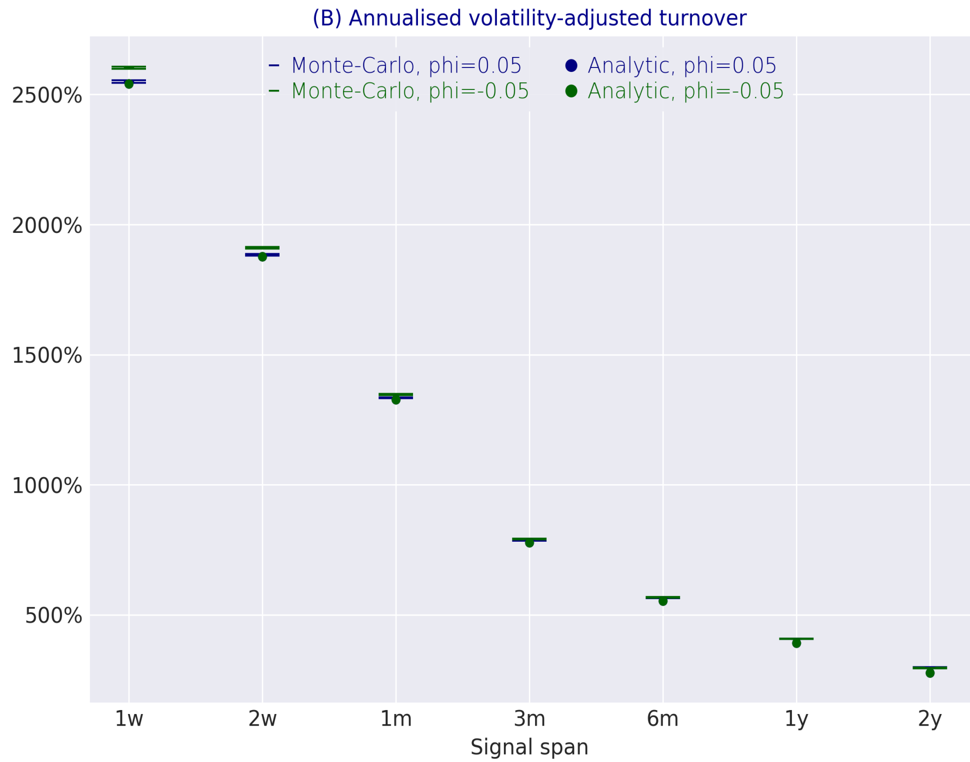
$$\bar{F}_{1y} = c_{1y}\left[\frac{\phi\nu}{1-\phi\nu}\right] + \left(\frac{l\sigma_{target}}{\sqrt{a}}\right)\mu_{an}^2 \tag{24}$$

- Positive if $\phi > 0$ when markets are diverging
- Negative if $\phi < 0$ when markets are mean-reverting

## Expected return for AR-1 process with zero drift

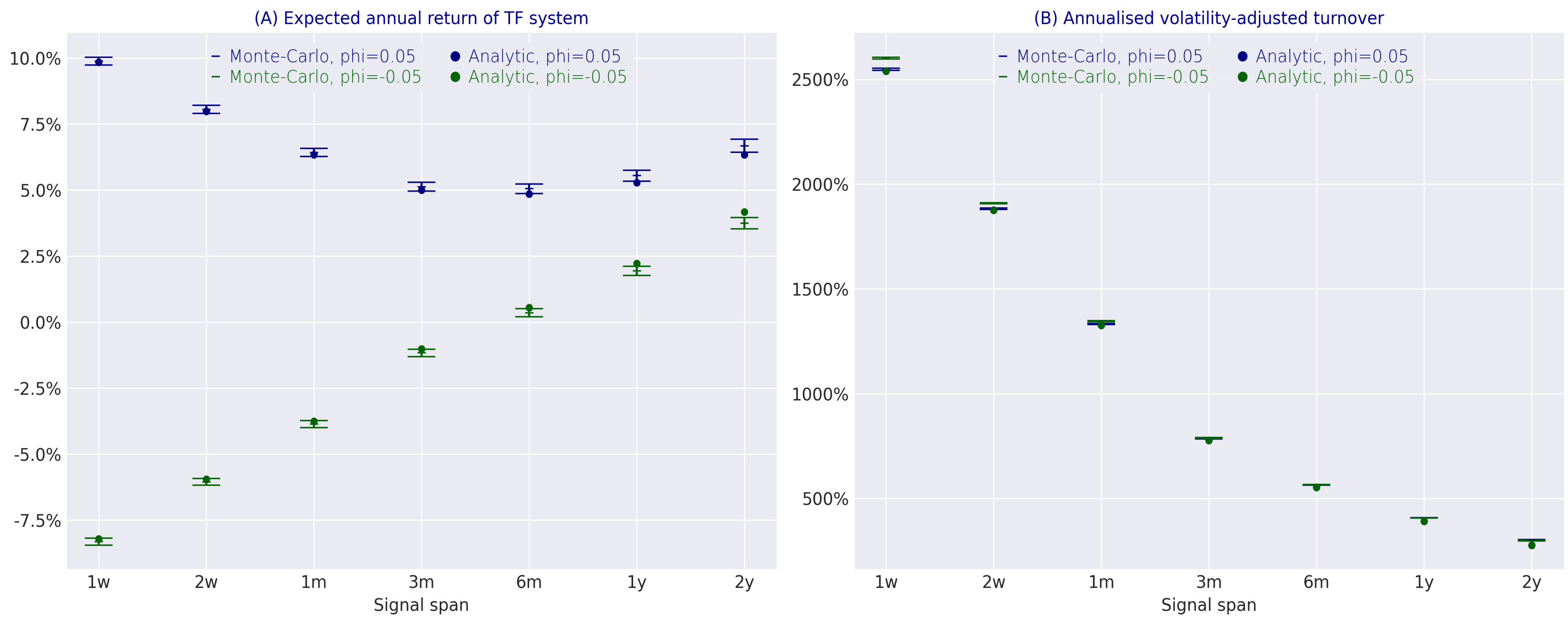
- Panel (A): Auto-correlation for $\phi \in \{0.05, -0.05\}$
- Panels (B) & (C): analytical values (dots) vs 95%



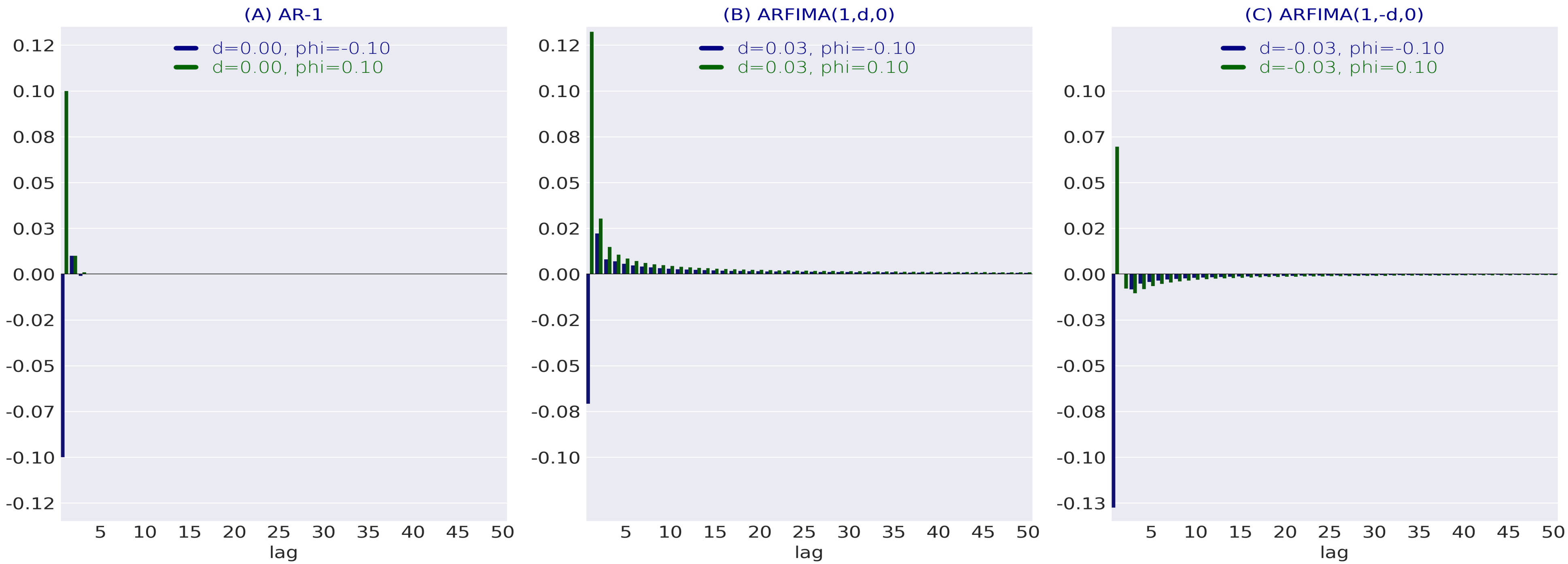
## Expected return for AR-1 with drift $\mu_{an} = 50\%$ (per $100\%$ vol)

- Negative mean-reversion at short periods is com-

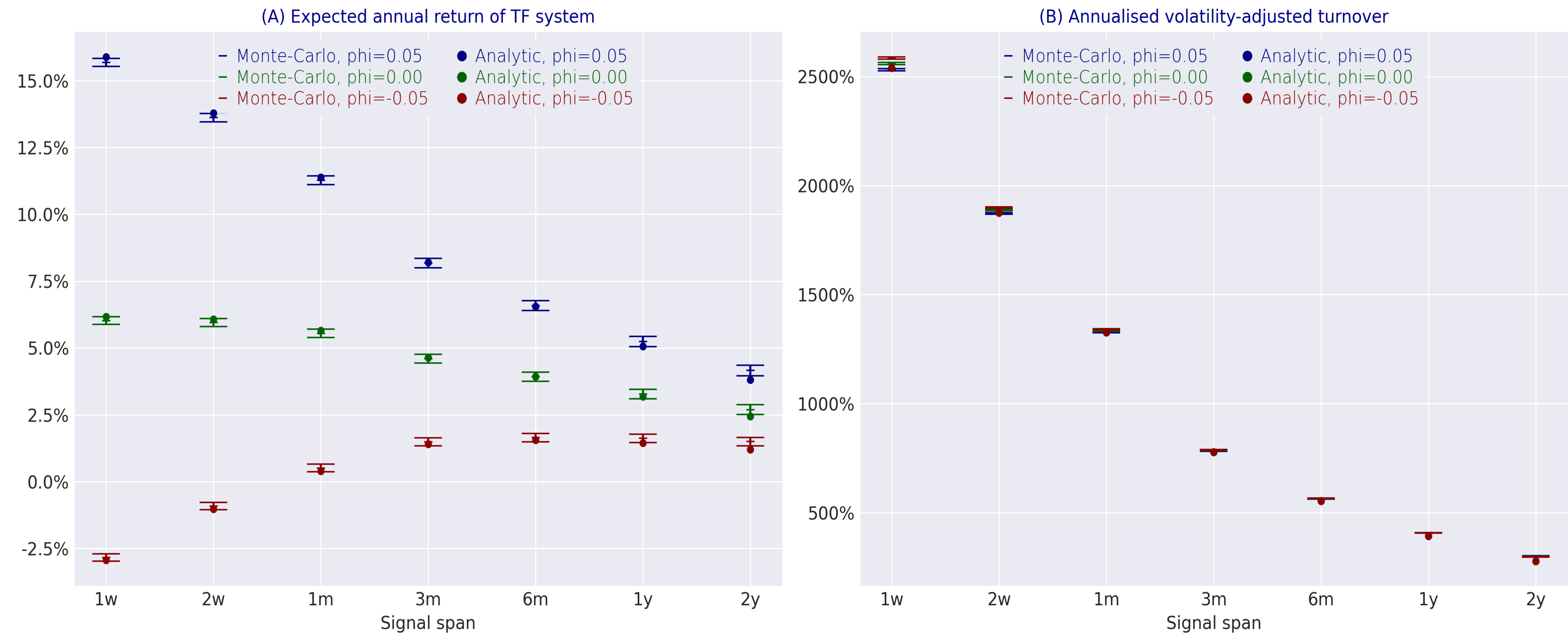


## Long Memory ARFIMA Processes [Granger1980], [Hosking1981]

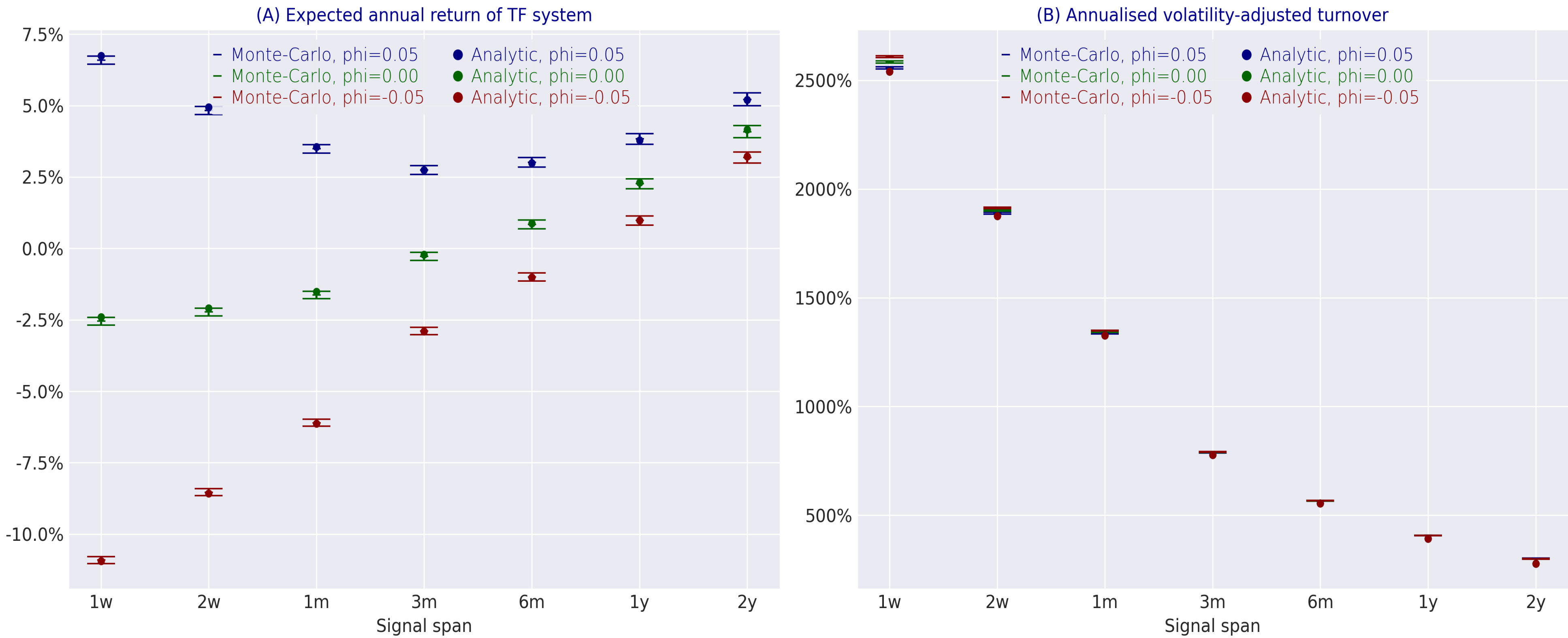
- ARFIMA $(0, d, 0)$ process is discrete-time analogue of fractional Gaussian noise with fractional order $d$
- Autocorrelation of $\tilde{r}_t$ is slowly decaying power:



# Expected return for ARFIMA $(1, d, 0)$ processes with long-term trend $d = 0.02$ and zero mean $\mu = 0.0$



## Expected return for ARFIMA $(1, d, 0)$ processes with long-term mean reversion with $d = -0.01$ and mean $\mu = 0.5$



**European TF System and Kelly ratio, I**

- Continuous-time dynamics for risky asset $s(t)$ and bank account $b(t)$ with rate $r$ under $\mathbb{P}$:

$$ds(t) = \mu s(t)dt + \sigma s(t)dW(t), \ b(t) = rb(t)dt \quad (25)$$

- The value $X(T)$ of portfolio with allocation weight $w$ to $s(t)$ and $1 - w$ to $b(t)$ at time $T$:

$$\frac{X(T)}{X(0)} = \exp\left\{\left(r + (\mu - r)w - \frac{1}{2}\sigma^2 w^2\right)T + \sigma w W(T)\right\}$$

- Consider maximisation of long-term growth:

$$\max_w \lim_{T\to\infty} \frac{1}{T}\mathbb{E}^{\mathbb{P}}\left[\log\left(e^{-rT}X(T)\right)\right] \quad (26)$$

- The solution is given by the Kelly ratio:

$$w^* = \frac{\mu - r}{\sigma^2} \quad (27)$$

**European TF System and Kelly ratio, II**

- Kelly ratio through Sharpe ratio & vol-target:

$$w^* = \mathbb{S} \times \frac{1}{\sigma}, \ \mathbb{S} = \frac{\mu - r}{\sigma} \tag{28}$$

- In reality, Sharpe ratios & vols are time-variate
- Instead, use estimates for two terms in Eq (29):

$$\widehat{w}^* = \hat{\mathbb{S}} \times \frac{1}{\widehat{\sigma}} \tag{29}$$

- For European TF, the weight $w_t = s_t \times v_t$ defined in Eq (15), Sharpe ratio is estimated using EWMA filter of risk-adjusted returns $z_t$ (EWMA of daily Sharpe ratios):

$$\hat{\mathbb{S}} = \mathcal{L}^{(\nu)}(z_t) \tag{30}$$

## Universe of Futures Markets

- Liquid contracts from 1962

| | Equities | Bonds | STIR | FX | Energy | Metals | Agriculture |
|---|---|---|---|---|---|---|---|
| 1 | S&P_500 | UST_2y | SOFR_3M | EURUSD | WTI | GOLD | CORN |
| 2 | NASDAQ | UST_5y | SONIA_3M | GBPUSD | BRENT | COPPER | WHEAT_SOFT |
| 3 | RUS2000 | UST_10y | EURIBOR_3M | CHFUSD | NYH_ULSD | SILVER | WHEAT_HRW |
| 4 | DJIA | UST_10y_ULTRA | AUS_90D | CADUSD | GASOLINE | PLATINUM | WHEAT_MINEAPOLIS |
| 5 | MSCI_EM | UST_BOND | CAD_3M | JPYUSD | GASOIL | PALLADIUM | WHEAT_MIL |
| 6 | ESTOXX50 | UST_ULTRA | | AUDUSD | ETHANOL | TSI_IRON_ORE | SOYBEANS |
| 7 | DAX | GER_2y | | NZDUSD | NAT_GAS | | SOYB_MEAL |
| 8 | CAC | GER_5y | | SEKUSD | | | SOYB_OIL |
| 9 | AEX | GER_10y | | NOKUSD | | | CANOLA |
| 10 | FTSE100 | GER_30y | | MXNUSD | | | RAPESEED |
| 11 | SMI | GILT | | | | | LIVE_CATTLE |
| 12 | OMX30 | JB_10Y | | | | | LEAN_HOGS |
| 13 | NIKKEI225 | OAT_10Y | | | | | FEEDER_CATTLE |
| 14 | TOPIX | CAN_10Y | | | | | SUGAR_#11 |
| 15 | ASX200 | AUS_10Y | | | | | WHITE_SUGAR |
| 16 | S&P/TSX60 | BTP_10Y | | | | | COFFEE_C |
| 17 | TAIEX | | | | | | ROBUSTA |
| 18 | HANG_SENG | | | | | | COCOA |
| 19 | HSCEI | | | | | | LONDON_COCOA |
| 20 | CHINA_A50 | | | | | | Cotton #2 |
| 21 | NIFTY_50 | | | | | | |

- Transaction costs per volume ([**Hurst2017**])

| Asset Class | Time Period | % Volume Costs |
|---|---|---|
| Equities | to 1992 | 0.34% |
| | 1993-2002 | 0.11% |
| | from 2003 | 0.06% |
| Bonds | to 1992 | 0.06% |
| | 1993-2002 | 0.02% |
| | from 2003 | 0.01% |
| Commodities | to 1992 | 0.58% |
| | 1993-2002 | 0.19% |
| | from 2003 | 0.10% |
| Currencies | to 1992 | 0.18% |
| | 1993-2002 | 0.06% |
| | from 2003 | 0.03% |

## Grid Backtest of European TF System

- Core parameters: long & short spans in days
- (A) Sharpe ratio 2000-2025YTD (no perf fees)
- (B) Bear Sharpe ratio is realised during bear periods of 60/40 portfolio
- (C) Annualised avg costs

(A) Sharpe ratio

| | short=5 | short=10 | short=20 | short=30 | short=40 | short=50 | short=75 | short=100 |
|---|---|---|---|---|---|---|---|---|
| long=20 | -0.15 | 0.04 | | | | | | |
| long=50 | 0.20 | 0.31 | 0.41 | 0.46 | 0.50 | | | |
| long=75 | 0.34 | 0.42 | 0.49 | 0.53 | 0.56 | 0.58 | | |
| long=100 | 0.43 | 0.50 | 0.55 | 0.59 | 0.61 | 0.62 | 0.65 | |
| long=175 | 0.58 | 0.62 | 0.66 | 0.67 | 0.68 | 0.68 | 0.68 | 0.68 |
| long=250 | 0.64 | 0.67 | 0.69 | 0.69 | 0.69 | 0.69 | 0.68 | 0.67 |
| long=375 | 0.66 | 0.68 | 0.69 | 0.68 | 0.68 | 0.67 | 0.64 | 0.62 |
| long=500 | 0.65 | 0.66 | 0.66 | 0.65 | 0.64 | 0.63 | 0.61 | 0.58 |

(B) Bear Sharpe ratio

| | short=5 | short=10 | short=20 | short=30 | short=40 | short=50 | short=75 | short=100 |
|---|---|---|---|---|---|---|---|---|
| long=20 | 0.33 | 0.37 | | | | | | |
| long=50 | 0.46 | 0.47 | 0.49 | 0.49 | 0.49 | | | |
| long=75 | 0.49 | 0.50 | 0.50 | 0.49 | 0.49 | 0.48 | | |
| long=100 | 0.51 | 0.51 | 0.50 | 0.49 | 0.48 | 0.47 | 0.44 | |
| long=175 | 0.51 | 0.50 | 0.48 | 0.46 | 0.44 | 0.43 | 0.40 | 0.37 |
| long=250 | 0.48 | 0.47 | 0.45 | 0.43 | 0.41 | 0.39 | 0.36 | 0.33 |
| long=375 | 0.43 | 0.41 | 0.39 | 0.37 | 0.35 | 0.33 | 0.30 | 0.27 |
| long=500 | 0.37 | 0.36 | 0.34 | 0.32 | 0.30 | 0.28 | 0.24 | 0.22 |

(C) Annualised avg cost

| | short=5 | short=10 | short=20 | short=30 | short=40 | short=50 | short=75 | short=100 |
|---|---|---|---|---|---|---|---|---|
| long=20 | 6.5% | 4.8% | | | | | | |
| long=50 | 4.4% | 3.3% | 2.6% | 2.2% | 2.0% | | | |
| long=75 | 3.7% | 2.8% | 2.2% | 2.0% | 1.8% | 1.7% | | |
| long=100 | 3.3% | 2.6% | 2.1% | 1.9% | 1.7% | 1.6% | 1.5% | |
| long=175 | 2.7% | 2.2% | 1.8% | 1.7% | 1.6% | 1.5% | 1.5% | 1.4% |
| long=250 | 2.4% | 2.0% | 1.7% | 1.6% | 1.5% | 1.5% | 1.4% | 1.4% |
| long=375 | 2.1% | 1.8% | 1.6% | 1.5% | 1.5% | 1.5% | 1.4% | 1.4% |
| long=500 | 2.0% | 1.7% | 1.6% | 1.5% | 1.5% | 1.4% | 1.4% | 1.4% |

## Grid Backtest of American TF System

- Core parameters: spans of long/short filters

- Entry-point width $q$ & stop-loss width $p$: $p = q = 5.0$

(A) Sharpe ratio

| | short=5 | short=10 | short=20 | short=30 | short=40 | short=50 | short=75 | short=100 |
|---|---|---|---|---|---|---|---|---|
| long=20 | 0.24 | 0.41 | | | | | | |
| long=50 | 0.41 | 0.43 | 0.24 | 0.43 | 0.01 | | | |
| long=75 | 0.48 | 0.57 | 0.52 | 0.51 | 0.25 | 0.32 | | |
| long=100 | 0.55 | 0.66 | 0.72 | 0.67 | 0.53 | 0.52 | 0.21 | |
| long=175 | 0.69 | 0.75 | 0.77 | 0.77 | 0.77 | 0.76 | 0.58 | 0.45 |
| long=250 | 0.67 | 0.70 | 0.71 | 0.77 | 0.73 | 0.72 | 0.73 | 0.69 |
| long=375 | 0.67 | 0.73 | 0.75 | 0.78 | 0.78 | 0.77 | 0.78 | 0.72 |
| long=500 | 0.69 | 0.74 | 0.75 | 0.75 | 0.74 | 0.74 | 0.72 | 0.70 |

(B) Bear Sharpe ratio

| | short=5 | short=10 | short=20 | short=30 | short=40 | short=50 | short=75 | short=100 |
|---|---|---|---|---|---|---|---|---|
| long=20 | 0.14 | -0.06 | | | | | | |
| long=50 | 0.32 | 0.24 | 0.06 | 0.09 | 0.16 | | | |
| long=75 | 0.37 | 0.37 | 0.24 | 0.20 | 0.10 | 0.01 | | |
| long=100 | 0.47 | 0.48 | 0.37 | 0.28 | 0.22 | 0.18 | 0.08 | |
| long=175 | 0.55 | 0.53 | 0.50 | 0.44 | 0.37 | 0.32 | 0.22 | 0.19 |
| long=250 | 0.56 | 0.55 | 0.50 | 0.49 | 0.43 | 0.38 | 0.31 | 0.24 |
| long=375 | 0.45 | 0.47 | 0.43 | 0.40 | 0.39 | 0.36 | 0.28 | 0.19 |
| long=500 | 0.43 | 0.41 | 0.38 | 0.35 | 0.32 | 0.29 | 0.21 | 0.15 |

(C) Annualised avg cost

| | short=5 | short=10 | short=20 | short=30 | short=40 | short=50 | short=75 | short=100 |
|---|---|---|---|---|---|---|---|---|
| long=20 | 0.9% | 0.0% | | | | | | |
| long=50 | 3.0% | 2.0% | 0.7% | 0.1% | 0.0% | | | |
| long=75 | 3.2% | 2.5% | 1.6% | 1.0% | 0.5% | 0.1% | | |
| long=100 | 3.2% | 2.5% | 1.9% | 1.5% | 1.1% | 0.7% | 0.1% | |
| long=175 | 2.6% | 2.1% | 1.6% | 1.4% | 1.3% | 1.2% | 1.1% | 0.8% |
| long=250 | 2.2% | 1.7% | 1.4% | 1.2% | 1.1% | 1.0% | 0.9% | 0.9% |
| long=375 | 1.8% | 1.4% | 1.1% | 0.9% | 0.9% | 0.8% | 0.7% | 0.7% |
| long=500 | 1.4% | 1.1% | 0.9% | 0.8% | 0.7% | 0.7% | 0.6% | 0.5% |

## Grid Backtest of TSMOM TF System

- Core parameters: number of returns $L$ in one period and number of periods $M$ for aggregate signal

(A) Sharpe ratio

| | L returns=3 | L returns=6 | L returns=9 | L returns=13 | L returns=16 | L returns=20 | L returns=23 | L returns=27 |
|---|---|---|---|---|---|---|---|---|
| M periods=3 | -0.66 | 0.08 | 0.14 | 0.37 | 0.66 | 0.68 | 0.71 | 0.69 |
| M periods=6 | -0.17 | 0.38 | 0.63 | 0.77 | 0.68 | 0.74 | 0.76 | 0.80 |
| M periods=9 | -0.00 | 0.61 | 0.82 | 0.73 | 0.76 | 0.88 | 0.84 | 0.84 |
| M periods=13 | 0.34 | 0.74 | 0.79 | 0.80 | 0.81 | 0.83 | 0.63 | 0.75 |
| M periods=16 | 0.49 | 0.65 | 0.79 | 0.78 | 0.89 | 0.71 | 0.69 | 0.61 |
| M periods=20 | 0.54 | 0.77 | 0.86 | 0.84 | 0.73 | 0.72 | 0.62 | 0.54 |
| M periods=23 | 0.58 | 0.77 | 0.83 | 0.72 | 0.74 | 0.62 | 0.53 | 0.42 |
| M periods=27 | 0.69 | 0.84 | 0.88 | 0.75 | 0.68 | 0.55 | 0.42 | 0.38 |

(B) Bear Sharpe ratio

| | L returns=3 | L returns=6 | L returns=9 | L returns=13 | L returns=16 | L returns=20 | L returns=23 | L returns=27 |
|---|---|---|---|---|---|---|---|---|
| M periods=3 | 0.14 | 0.24 | 0.29 | 0.32 | 0.30 | 0.24 | 0.26 | 0.26 |
| M periods=6 | 0.23 | 0.32 | 0.39 | 0.32 | 0.18 | 0.14 | 0.17 | 0.15 |
| M periods=9 | 0.29 | 0.38 | 0.36 | 0.25 | 0.14 | 0.11 | 0.08 | 0.09 |
| M periods=13 | 0.35 | 0.36 | 0.29 | 0.19 | 0.07 | 0.05 | -0.02 | 0.00 |
| M periods=16 | 0.42 | 0.30 | 0.23 | 0.13 | 0.09 | -0.01 | -0.03 | -0.07 |
| M periods=20 | 0.37 | 0.29 | 0.19 | 0.10 | 0.01 | -0.06 | -0.07 | -0.13 |
| M periods=23 | 0.35 | 0.25 | 0.14 | 0.05 | -0.00 | -0.09 | -0.14 | -0.20 |
| M periods=27 | 0.39 | 0.24 | 0.13 | 0.05 | -0.06 | -0.16 | -0.21 | -0.24 |

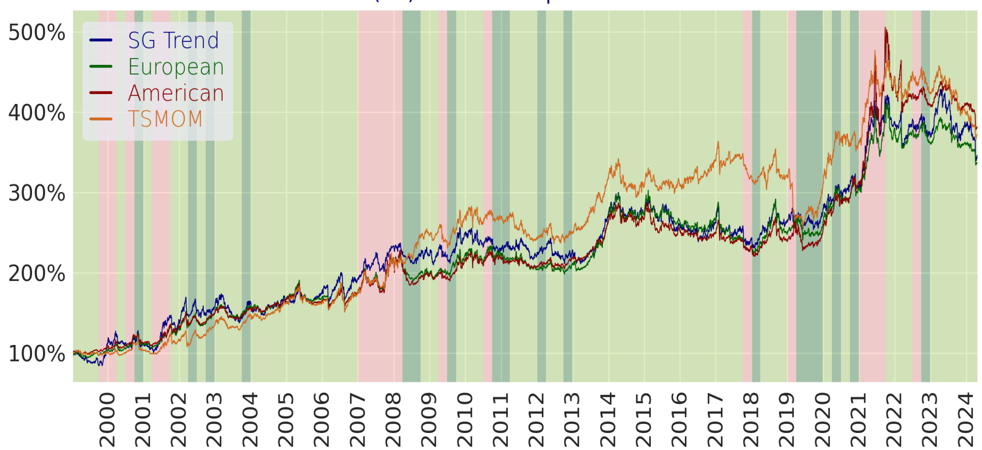
(C) Annualised avg cost

| | L returns=3 | L returns=6 | L returns=9 | L returns=13 | L returns=16 | L returns=20 | L returns=23 | L returns=27 |
|---|---|---|---|---|---|---|---|---|
| M periods=3 | 9.5% | 4.8% | 3.3% | 2.3% | 1.8% | 1.5% | 1.3% | 1.1% |
| M periods=6 | 6.8% | 3.4% | 2.4% | 1.7% | 1.4% | 1.1% | 1.0% | 0.9% |
| M periods=9 | 5.6% | 2.9% | 2.0% | 1.4% | 1.2% | 1.0% | 0.9% | 0.8% |
| M periods=13 | 4.7% | 2.4% | 1.7% | 1.3% | 1.1% | 0.9% | 0.8% | 0.7% |
| M periods=16 | 4.3% | 2.3% | 1.6% | 1.2% | 1.0% | 0.8% | 0.8% | 0.7% |
| M periods=20 | 3.9% | 2.1% | 1.5% | 1.1% | 0.9% | 0.8% | 0.7% | 0.7% |
| M periods=23 | 3.6% | 1.9% | 1.4% | 1.1% | 0.9% | 0.8% | 0.7% | 0.7% |
| M periods=27 | 3.4% | 1.8% | 1.3% | 1.0% | 0.9% | 0.8% | 0.7% | 0.6% |

## Base Comparison with SG Trend Index from 2000 to 2025YTD

- 2/20% management/performance fees
- Background colours: 16%/68%/16% worst/mid/be quarters of 60/40 Equity/Bond portfolio

(A1) Cumulative performance




(A2) Risk-adjusted performance table

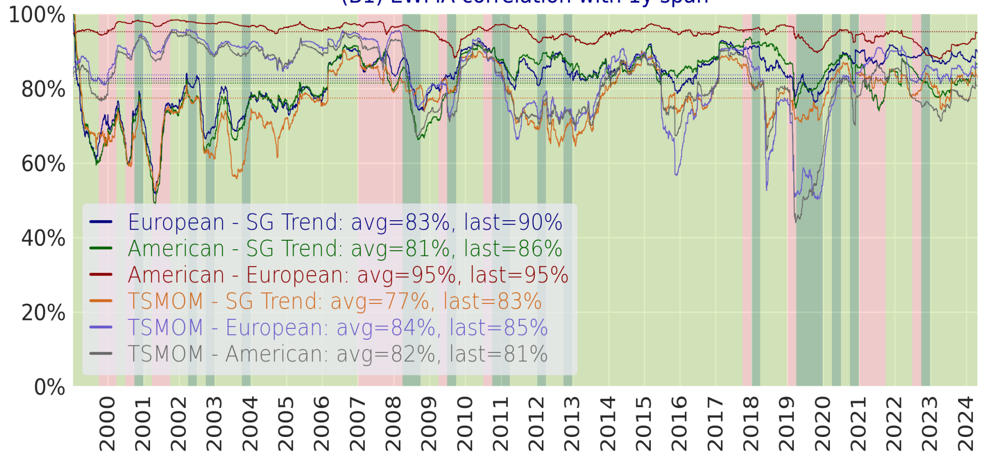
| Assets | P.a. return | Vol | Sharpe (rf=( | Max DD | Skewness | An Alpha | Beta | R2 |
|---|---|---|---|---|---|---|---|---|
| 60/40 Equity/Bond | 4.5% | 9.1% | 0.50 | -34% | -0.59 | -0.0% | 1.00 | 100% |
| SG Trend | 5.0% | 13.4% | 0.37 | -23% | 0.00 | 6.6% | -0.14 | 1% |
| European | 4.9% | 11.3% | 0.44 | -26% | 0.03 | 7.2% | -0.30 | 6% |
| American | 5.4% | 11.3% | 0.48 | -25% | 0.41 | 7.9% | -0.36 | 8% |
| TSMOM | 5.3% | 11.3% | 0.47 | -30% | -0.24 | 5.9% | 0.01 | 0% |

(B2) Annual returns

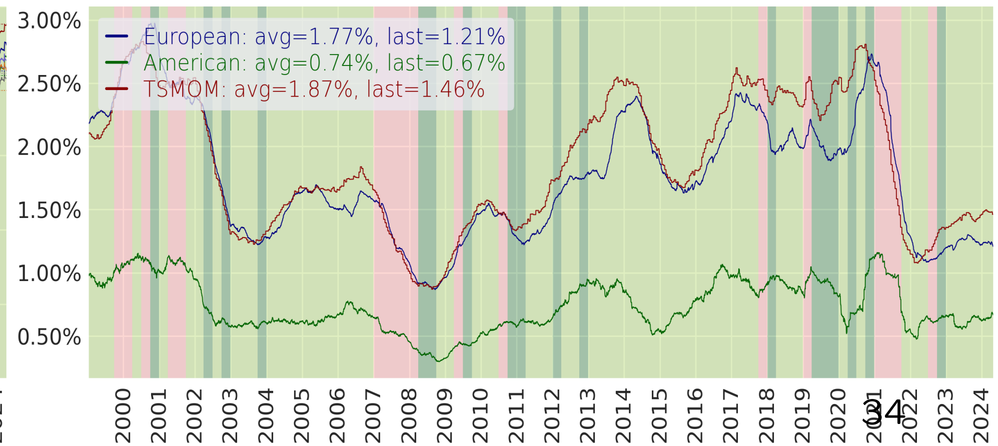
| | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| SG Trend | 12% | -0% | 26% | 12% | 3% | 1% | 8% | 9% | 21% | -5% | 13% | -8% | -4% |
| European | 3% | 7% | 20% | 13% | 6% | 6% | 1% | 3% | 23% | -8% | 13% | 0% | -9% |
| American | 7% | 5% | 20% | 11% | 4% | 7% | -1% | 6% | 21% | -7% | 11% | 1% | -5% |
| TSMOM | -0% | 4% | 13% | 14% | 10% | 10% | 0% | 7% | 22% | 15% | 10% | -1% | -10% |

| | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 | 2019 | 2020 | 2021 | 2022 | 2023 | 2024 | 2025 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| SG Trend | 3% | 20% | 0% | -6% | 2% | -8% | 9% | 6% | 9% | 27% | -4% | 3% | -10% |
| European | 3% | 26% | 2% | -4% | 2% | -13% | 7% | 6% | 15% | 26% | -5% | -1% | -6% |
| American | 5% | 21% | 1% | -6% | -1% | -7% | 3% | 7% | 22% | 43% | -8% | 0% | -7% |
| TSMOM | 3% | 28% | -3% | -0% | 10% | -6% | -0% | -2% | 16% | 23% | -5% | -6% | -6% |

(B1) EWMA correlation with 1y span

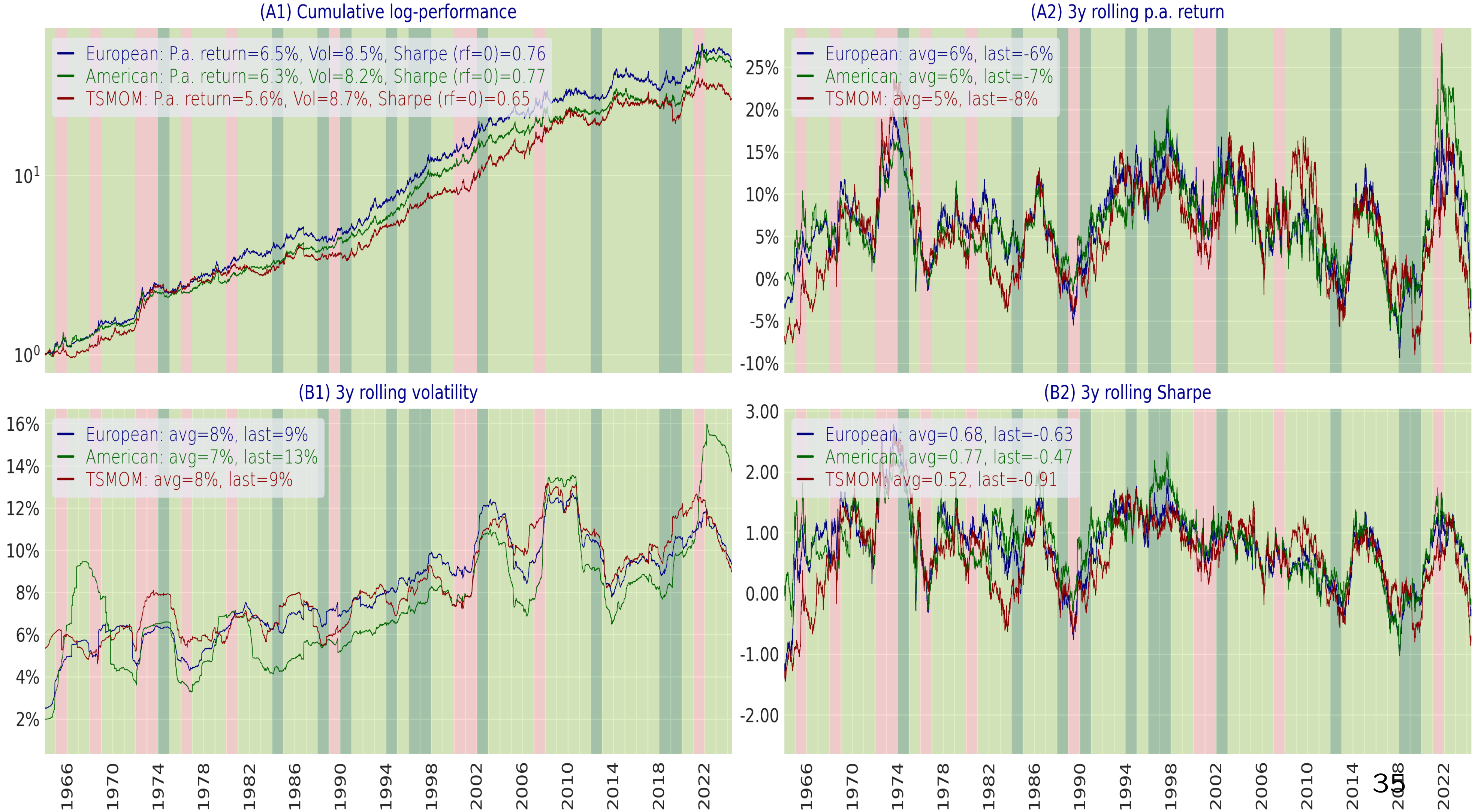



(C2) 1y rolling cost



## Long-Term Backtest from 1965

- Systems vol dependent on number of futures
- Robust perf in "red" years of 60/40 portfolio



**Portfolio Volatility Targeting**

- Portfolio volatility for current weights $w$:

$$v_t = \sqrt{w^T \Sigma_t w} \tag{31}$$

- $\Sigma_t$ is covariance matrix estimated using EWMA model with a given lookback span

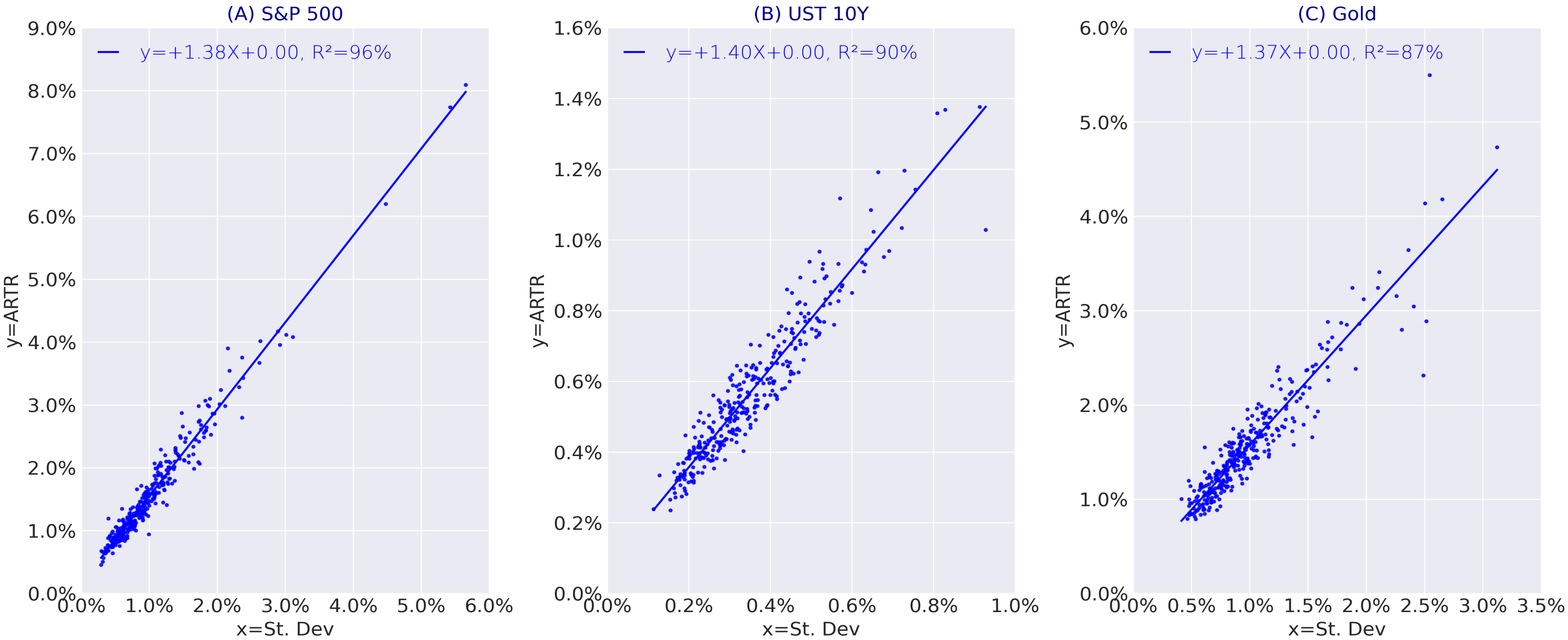
- All positions are rescaled using ratio $R = \frac{v_{target}}{v_t}$

- **Pro:** 1) risk-budget is fully utilized 2) return profile is smoother

- **Cons:** Positive skeweness of TF returns?

**ATR vs Volatility for Position Sizing: Empirical analysis**

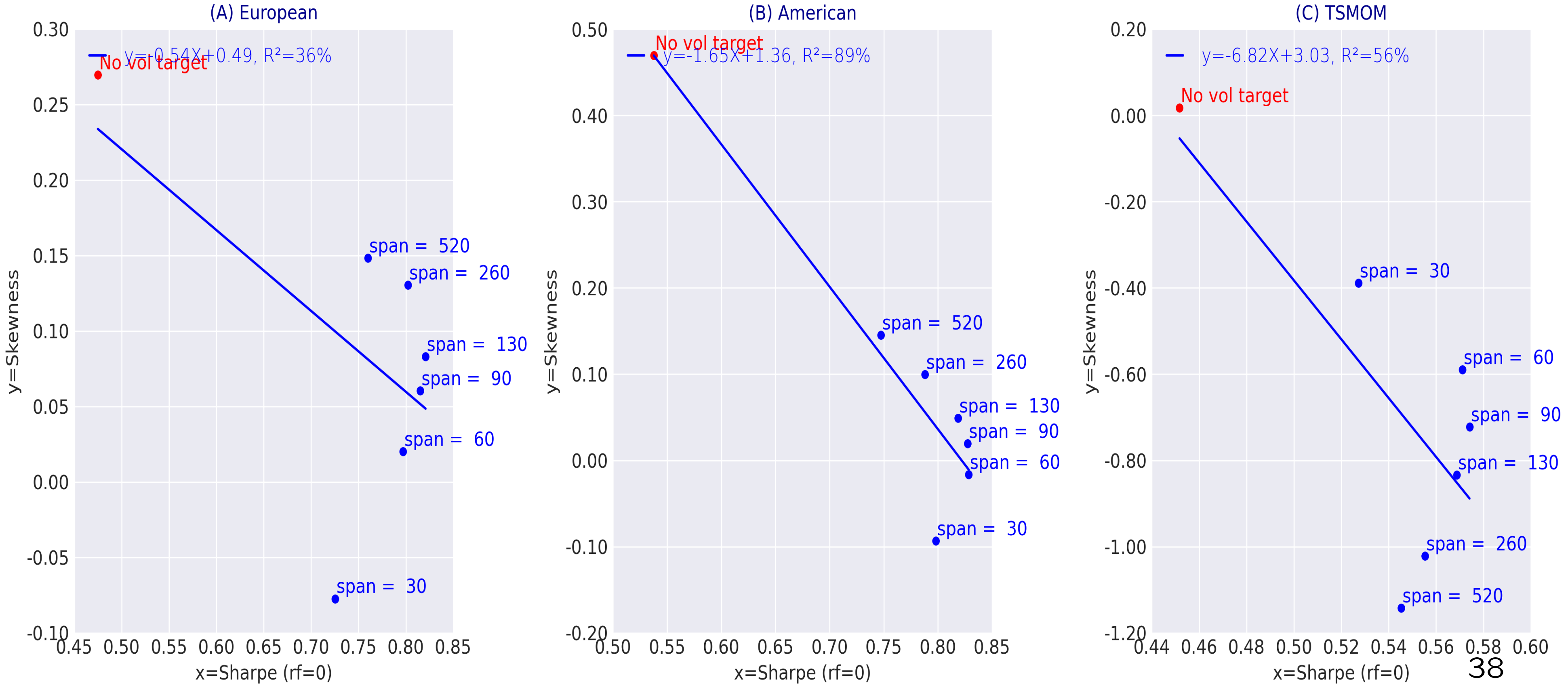
- European TF systems scale exposure by $1/\sigma_t$
- American TF systems scale exposure by $1/ATR_t$
- (A) & (B): monthly ATR vs realised vol for S&P500 and UST 10Y future: $\beta \approx 1$ with $R^2 = 90\%$



**Portfolio Vol-Targeting and Skeweness**

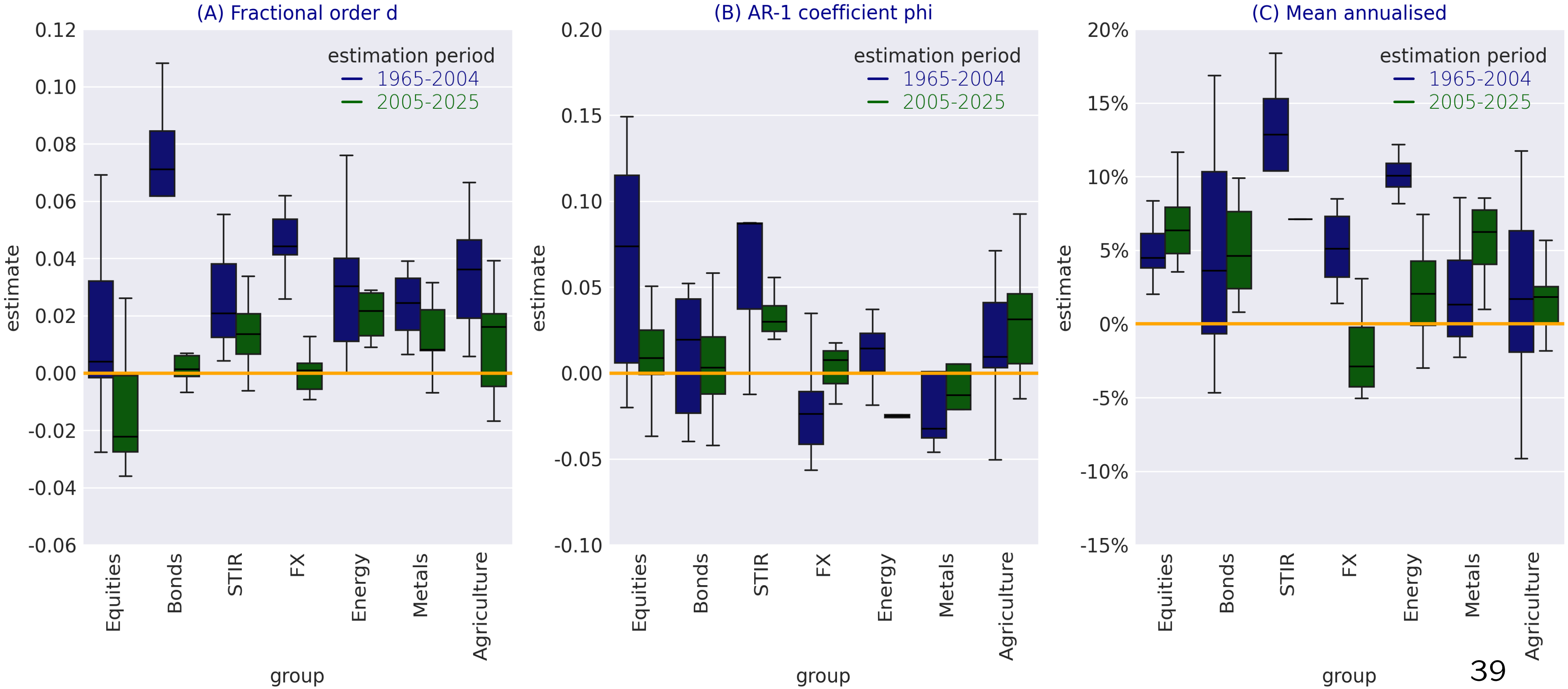
- Apply Eq 31 for portfolio vol-targeting with span for EWMA covariance from 30 to 260 days

- Skeweness of TF quarterly returns (y-axis) vs Sharpe (x-axis) from 2000 to 2025YTD



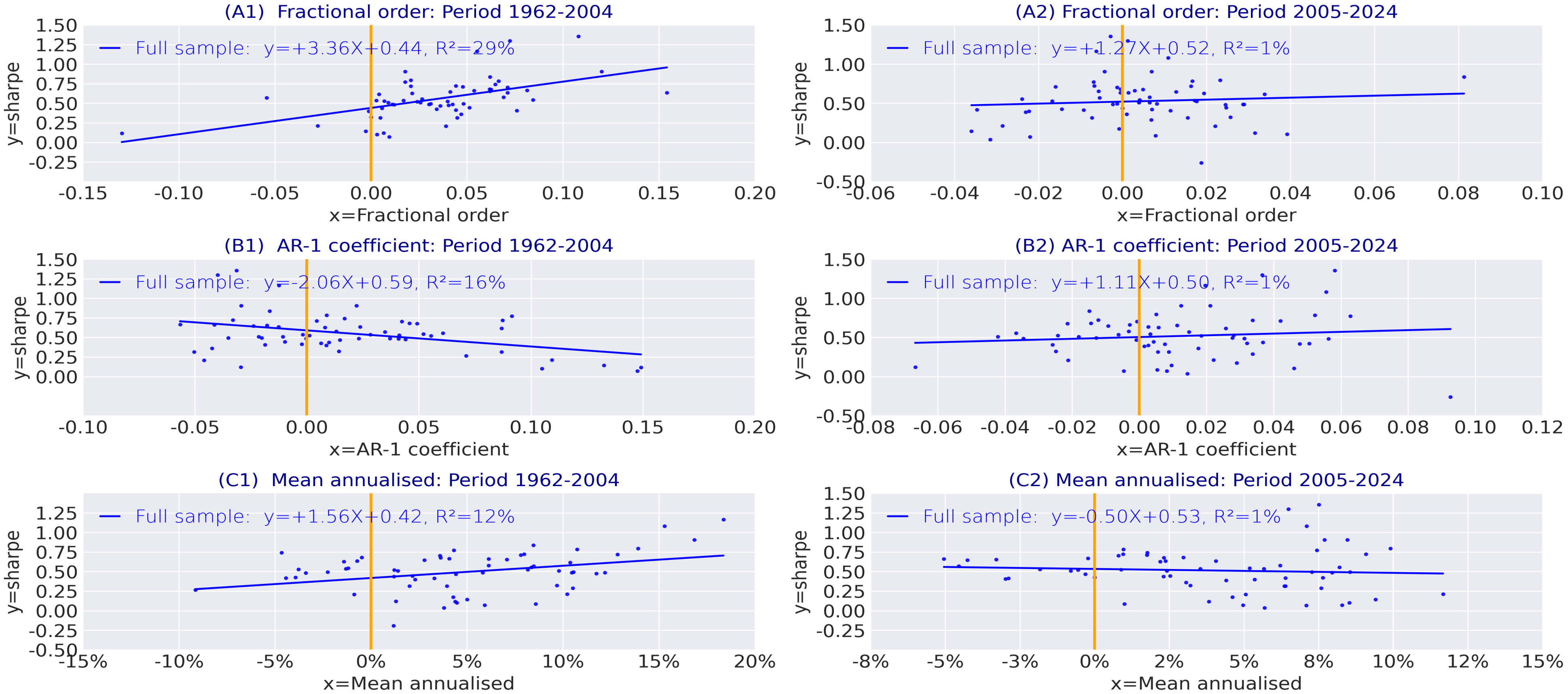
**Empirical analysis using ARFIMA $(1, d, 0)$ model**

- Boxplot of estimated ARFIMA for 1962-2004 & 2004-2024 aggregated by asset classes (A): fractional order, (B): mean-reversion, (C): drift
- 1962-2004 period: stronger long memory feature



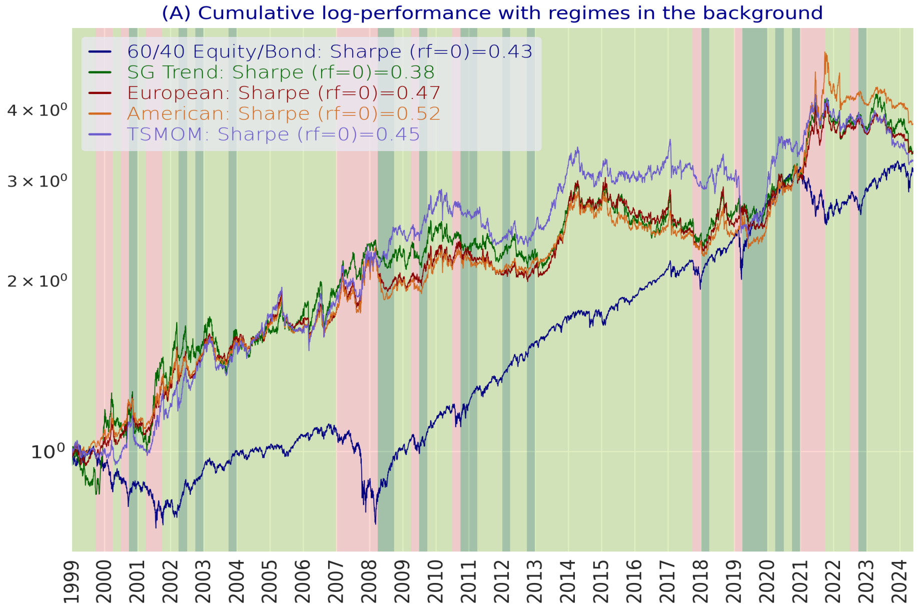
## Instrument returns vs parameters of ARFIMA $(1, d, 0)$ process

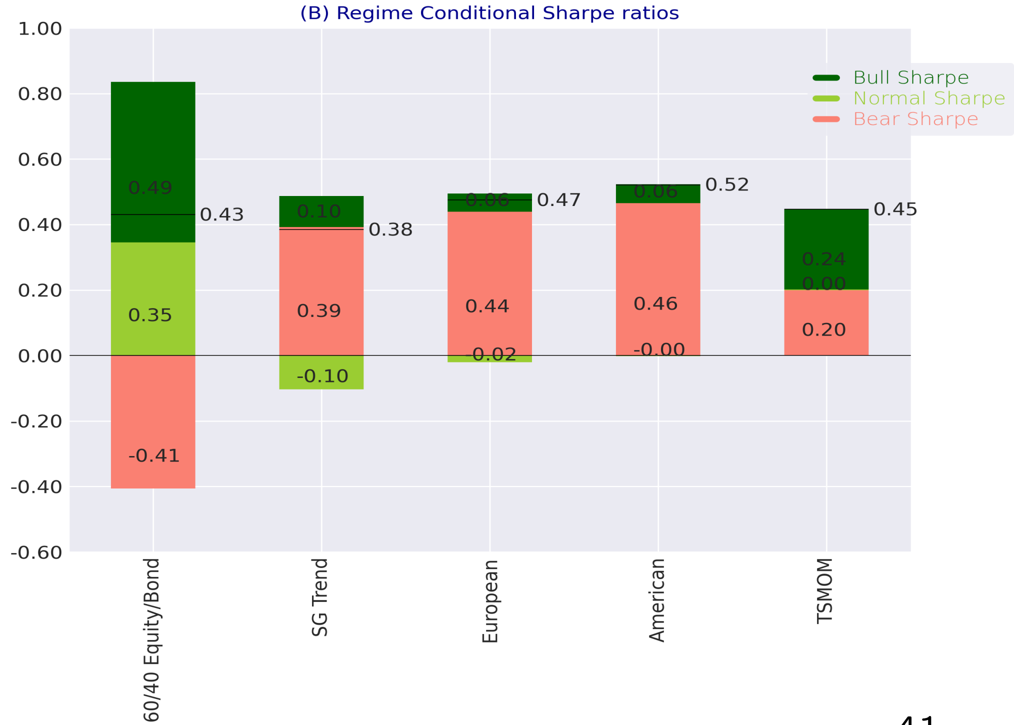
- Instrument Sharpe vs param estimate for (A) fractional order, (B) AR-1 coef, (C) drift
- 1962-2004: long memory feature is significant



## TF as Defensive Overlay for Long Portfolios [Sepp2019], [Sepp2020]

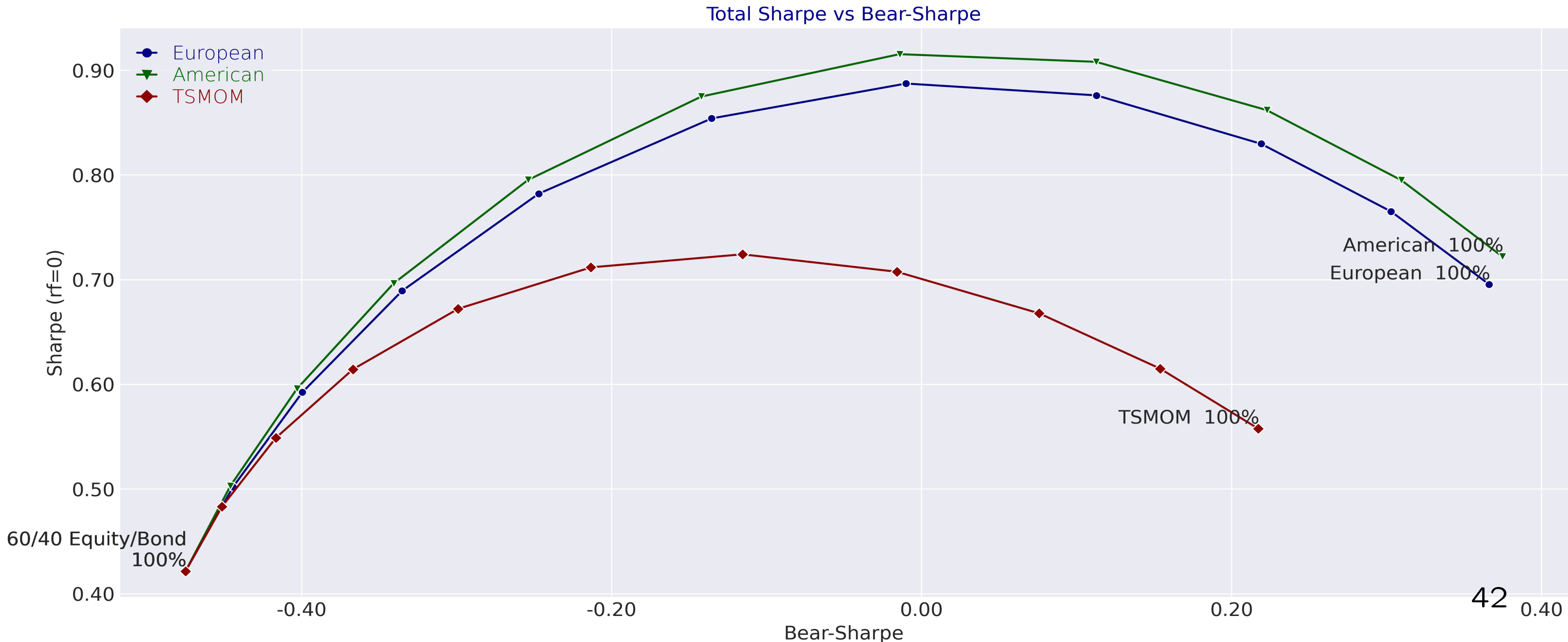
- (A) Identify Bear/Normal/Bull regimes of 60/40 Equity/Bond portfolio as 16%/68%/16% worst/mid/ quantiles of quarterly returns
- (B) : split total Sharpe ratio into regime-conditional Bear/Normal/Bull Sharpe ratios



## Smart Diversification of Long-only Portfolios

- Convex risk-reward frontier: x-axis is Bear Sharpe (portfolio risk) & y-axis is Total Sharpe ratio
- Optimal: 50%/50% Balanced/TF system
- 2x leverage of 50%/50% Balanced/TF system is know as returns stacking



**Conclusions: Design of trend-following systems**

1) Faster TF with focus on detection of short-term auto-regressive trends

2) Medium-term TF capturing trends due to long-memory

3) Long-term TF systems with focus on capturing long-term drift

- 3 methods to design TF systems with similar empirical performances
- Smart Diversification for long-only portfolios

**References**


• Trend-following, Risk-parity and the influence of Correlations (2015), in E. Jurczenko (ed.), Risk-based and Factor Investing, ISTE Press & Elsevier, chapter 3, pp. 65–96

• Baz J., Granger N., Harvey C.R., Le Roux N., Rattray S. (2015). Dissecting Investment Strategies in the Cross Section and Time Series, SSRN preprint

• Bernardo, J.M., Smith, A.F.M. (1994). Bayesian theory, Wiley.

• Bouchaud, J.P., Dau, T.L., Deremble, C., Lemprire, Y., Nguyen, (2017), Tail Protection for Long Investors: Convexity at Work. Journal of Investment Strategies, 7(1), 61–84.

• Faith C (2007). Way of the turtle, McGraw-Hill

• Granger CWJ, Joyeux R (1980). An introduction to long-memory time series models and fractional differencing. Journal of Time Series Analysis, 1(1), 15–29.

• Grebenkov D, Serror J (2014). Following a trend with an exponential moving average: Analytical results for a Gaussian model. Physica A: Statistical Mechanics and its Applications, 394, 288–303.

• Holt, C.C. (1957). Forecasting seasonals and trends by exponentially weighted moving averages. O.N.R. Memorandum 52/1957
• Hosking, JRM. (1981). Fractional differencing. Biometrika, 68(1), 165–176.
• Hurst, B., Y. H. Ooi, and L. H. Pedersen. (2017). A Century of Evidence on Trend-Following Investing. The Journal of Portfolio Management 44 (1), 15–29
• Hurst, B., Y. H. Ooi, and L. H. Pedersen. (2013). Demystifying managed futures, Journal of Investment Management 11(3), 42–58.
• Martin, R. J. (2023). Design and analysis of momentum trading strategies, Preprint `https://arxiv.org/abs/2101.01006`
• Moskowitz, T., Ooi, Y. H. and Pedersen, L. H. (2012). Time series momentum, Journal of Financial Economics 104(2), 228–250.
• Sepp A., and Dezeraud L. (2019). Trend-Following CTAs vs Alternative Risk-Premia: Crisis beta vs risk-premia alpha, The Hedge Fund Journal 138, 20–31
• Sepp A. (2020). 60/40 Portfolios and the Need for Smart Diversification, HedgeNordic, Systematic Strategies Special Report, June 2020, 24–29 `https://hedgenordic.com/wp-content/uploads/2020/06/systematic_strategi`

pdf
• Valeyre, S. (2025). Breaking the Trend: How to Avoid Cherry-Picked Signals. Working paper https://arxiv.org/abs/2504.10914
• Zakamulin, S., and Giner, J. (2020). Trend following with momentum versus moving averages: a tale of differences. Quantitative Finance 20(6), 985–1007

**Disclosure**

These slides and discussion represent my personal views.

These views do not represent an official view of my current and last employers.

These views and discussion are not an investment advice in any possible form.

Futures are associated with high risk.